Chapter 14
An Ancient Chinese Flat Earth Cosmology: Details and Calculations

Contents

Based on the introductory representation in the previous chapter, this chapter provides more details, calculations, and an extrapolation.

The Location of Zhou

According to the *Zhou bi*, the distance between Zhou and the pole, or more precisely, the subpolar point, is 103,000 *li*,[1] which equals 42,827.4 km. This absurd distance is the consequence of the shadow rule which says that 1 cun shadow more or less equals 1000 *li* north- or southward.[2] Cullen has tried to determine the actual latitude of Zhou. He obtains a value of 35.33°, which is, as he says, easy to calculate.[3] Cullen's easy calculation presupposes that the shadow measurements at the summer solstice (1.6 *chi*, cf. texts 13.18 and 13.19) and at the winter solstice (13.5 *chi*,

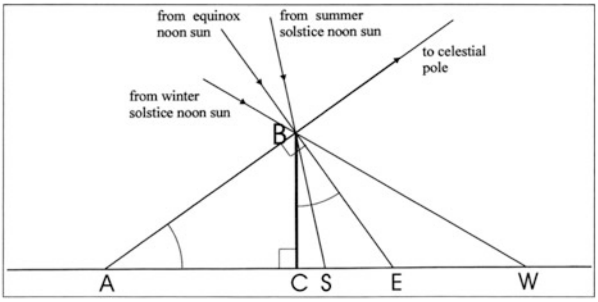
[1]Cf. texts 13.8 and 13.29 (#B14), 13.9 and 14.28 (#B33).

[2]Cf. texts 13.10 (#B10), 13.11 (#B12), and 13.12 (#D18).

[3]Cullen (1996), 104–105.

D. L. Couprie, *When the Earth Was Flat*, Historical & Cultural Astronomy,
https://doi.org/10.1007/978-3-319-97052-3_14

Fig. 14.1 Calculation of the latitude of Zhou (Adapted after Cullen 1996, 105, Fig. 10)



cf. texts 13.23–13.25) with a gnomon of 8 *chi*, were based on actual observations.[4] The calculation goes as follows (see Fig. 14.1): the angle between the gnomon BC and the sunray at the summer solstice (∠CBS) = $\tan^{-1}$ (1.6/8) ≈ 11.31°. The angle between the gnomon and the sun ray at the winter solstice (∠CBW) = $\tan^{-1}$ (13.5 ÷ 8) ≈ 59.35°. The angle between the sunray at the summer solstice and the sunray at the winter solstice (∠SBW) = ∠CBW – ∠CBS = 59.35° – 11.31° = 48.04°. The equinox sunray is the bisector of ∠SBW, so ∠SBE = 48.04° ÷ 2 = 24.02°. ∠CBE, which is the sum of ∠CBS and ∠SBE, = 11.31° + 24.02° = 35.33°. Since Δ BCE and Δ ABE are similar, ∠CBE = ∠BAE = 35.33°, which is the latitude of Zhou.

To an angle of 35.33° belongs a shadow CE of 8 ÷ tan (90 − 35.33) = 5 *chi* 6 *cun* 7 *fen*, which differs from the 7 *chi* 5 *cun* 5 *fen* in the *Zhou bi* (cf. texts 13.26–9.128). Trying to calculate the latitude of Zhou with the help of the length of the shadow at the equinoxes according to the text of the *Zhou bi* results in a latitude for Zhou of $\tan^{-1}$ (7.55 ÷ 8) ≈ 43.34°. The cause of this mistake is that out of the 24 shadow lengths in the table of #H2 only those of the summer and winter solstices were (more or less) based on observation, while the other values were not calculated but interpolated are equal distances between the measured shadow lengths of the gnomon at noon at the winter and summer solstices.[5] We already saw this in the previous chapter, Sect. *The Shadow Rule and the Fundamental Cosmic Measurements*, for the shadow length at noon during the solstices. In the table of #H2, the shadow rule is not expressed as "one *chi* for a thousand *li*", but by the assumption that the shadow length of the gnomon over the year from one winter solstice (when the shadow equals 13.5 *cun*) until the next summer solstice (when the shadow equals 13.5 *cun*) shortens 12 times with the same amount of

[4]The differences between the actual figures for the shadow lengths at the solstices and those of the *Zhou bi* are only slight and could be due to the inaccuracy of the measurements. At the summer solstice, the maximum altitude of the sun at 35.33° N was 78.2° and the shadow length 8 ÷ tan 78.2 = 1 *chi* 6 *cun* 7 *fen* (according to the *Zhou bi*: 1 *chi* 6 *cun*); at the winter solstice, the minimum altitude of the sun at 35.33° N was 59.3° and the shadow length 8 ÷ tan 59.3 = 1 *zhang* 3 *chi* 4 *cun* 7 *fen* (according to the *Zhou bi*: 1 *zhang* 3 *chi* 5 *cun*).

[5]Cf. Cullen (1996), 196: "[the] sole purpose [of this section #H] is to provide a list of noon gnomon shadows for each of the 24 *qi* throughout the year. This is done by linear interpolation between the values for the summer and winter solstices, which are the only data to bear any close relation to observation."

9 *cun*, 9 *fen* and 1/6 *fen*, and then grows again 12 times with an equal amount until the next winter solstice. This version of the shadow rule is formulated in the following lines of section H of the *Zhou bi*:

14.1. (#H1) For the 24 *qi* (. . .), the decrease or increase [of the shadow] for one *qi* is 9 *cun* 9 *fen* and 1/6 *fen*. The length of the winter solstice shadow is 1 *zhang* 3 *chi* 5 *cun*, and the length of the summer solstice shadow is 1 *chi* 6 *cun*.
14.2. (#H3) For the 24 *qi* (. . .), the decrease or increase [of the shadow] for one *qi* is 9 *cun* 9 *fen* and 1/6 *fen*. The winter and summer solstices are the beginnings of decrease and increase.
14.3. (#H4) Method:
Set up the winter solstice shadow, subtract the summer solstice shadow, and the difference is made the dividend.
Take 12 as the divisor.
The integral quotient gives the cun.
Multiply the remainder by ten and divide again to obtain the fen.
Make the [final] remainder the numerator over the determinator.

The text of 14.3 (#H4) is somewhat cryptic. First of all, "dividend" and "numerator" are interchangeable terms, and so are "divisor" and "determinator." The first line, expressed in *cun*, results in $135 - 16 = 119$ *cun*. Divided by 12 this makes about 9.917 *cun* in our decimal notation. The term "integral quotient" refers to the integer of the quotient (which is, 9 *cun*). "The remainder" should be taken to mean the quotient minus the integer. This remainder multiplied by ten gives 9.17, of which the integer is 9 *fen*. The expression "divide again" is somewhat obscure and should be understood as something like "take the integer again to obtain the *fen*." "The final remainder" should be taken to mean the number 17, and "making it the numerator over the determinator" something like "divide it by 100", which makes 17/100 or 1/6 *fen*.

When we try to measure the latitude of Zhou by means of the "polar shadow" of 10.3 *chi* (see #B14 in text 13.29), the result is $\tan^{-1}(80{,}000 \div 103{,}000) \approx 37.84°$, whereas to a latitude of 35° belongs a length of the "polar shadow" of 11.4 *chi* ($8 \div \tan 35 \approx 11.4$), as Cullen points out.[6] The distance from the pole to the point in the zenith of Zhou should therefore be 114,000 *li* instead of the 103,000 *li* given by the *Zhou bi*. Cullen's conclusion is that the Chinese somewhat cheated to get "a neatly arranged universe in which the summer and winter paths were related in size by a factor of two," even taken into account that "the celestial pole cannot in any case be observed directly."[7]

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[6]See Cullen (1996), 106.
[7]Ibidem.

Measuring the Sun's Diameter

In the *Zhou bi*, not only the distance to the sun is measured, as indicated in the previous chapter,[8] but also the sun's diameter. Two essential distinctions must be kept in mind to ensure a proper understanding of the following. First, the angular diameter versus the actual diameter of the sun. The angular diameter is the angle of an object as seen by an observer. The angular diameter of the sun is about 0.5°. The real diameter of the sun is 1392 million km, but for the ancient Chinese, who believed that the earth is flat and therefore the sun is not at an enormous distance, it must have been much smaller. Second, the oblique distance versus the vertical distance of the sun. The oblique distance of the sun is the distance from an observer to the sun. The vertical distance is the distance from the flat earth to the flat heaven, which according to the *Zhou bi* is always 80,000 *li*. The measurement of the size of the sun in the *Zhou bi* sun is otherwise than that discussed in Chaps. 3 and 11. Since in the *gai tian* system, the sun does not turn around the earth but parallel to the earth's flat surface, the Chinese could not use the distance from the observer to the sun at the summer solstice as the radius of the sun's orbit. So, they used this distance in another, ingenious way, by using a sighting tube:

14.4. (#B11) Wait until the base is six *chi* in length, then take a bamboo [tube] of diameter one *cun*, and of length eight *chi*. Catch the light [down the tube] and observe it: the bore exactly covers the sun, and the sun fits into the bore. Thus it can be seen that an amount of eighty *cun* [=8 *chi*] gives one cun of diameter. (. . .) The oblique distance to the sun from the position of the *bi* is 100,000 *li*. Working things out in proportion, eighty *li* gives one *li* of diameter, thus 100,000 *li* gives 1250 *li* of diameter. So, we can state that the diameter of the sun is 1250 *li* (Fig. 14.2).

The oblique distance of 100,000 *li* from the observer to the sun was calculated using a gnomon of 8 *chi* (80 *cun*) with a shadow length of 6 *chi*, as shown in Fig. 13.4. The calculation of the diameter of the sun is a simple equation: 80: 1 = 100,000: x → x = 1250. Converted in kilometers, the diameter of the sun is: $1250 \times 0.4158 = 519.75$ km.[9] With regard to this calculation of the sun's diameter, Cullen notes: "The figures given here predict an apparent solar diameter of 43′, which is about 10′ greater than the value actually observed."[10] The calculation behind this remark is that when we take 100,000 *li* as the radius of a circle with the observer as its center, the circumference of this circle $= 2\pi \times 100{,}000 = 600{,}000$ *li*.[11] The apparent diameter of the sun is about

[8]See Figs. 13.4 and 13.5 and the accompanying text.

[9]The calculation given in Couprie (2011), 198–199 and Fig. 16.6 is needlessly complicated and partly makes use of wrong numbers, e.g. for the oblique distance of the sun, although the right number for the diameter of the sun is mentioned on p. 198.

[10]Cullen (1996), 128.

[11]Taking $\pi = 3$.

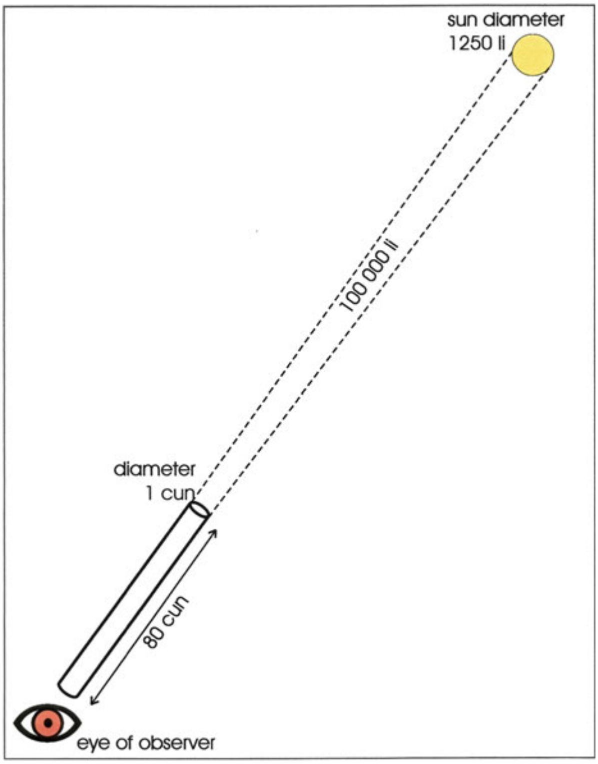

Fig. 14.2 The sighting tube and the diameter of the sun (not to scale) (In Couprie 2011, 199, Fig. 16.6, I needlessly complicated this picture, calculating with similar triangles)

0.5°, which would result in a diameter of the sun of 600,000 ÷ 720 ≈ 833 *li*. By the measurements with the sighting tube in the *Zhou bi*, however, the diameter is 1250 *li*. The apparent diameter of the sun belonging to that number is (1250 ÷ 873) × 0.5 ≈ 0.716°; or expressed in minutes, as Cullen does: 0.716 × 60 ≈ 43′. According to Cullen, this difference must be due to the deliberately sized sighting tube, whose length is exactly equal to the length of the standard gnomon and its bore exactly one *cun*.[12] This means that the Chinese astronomers consciously chose the dimensions of the sighting tube in order to bring them into line with their calculations of the celestial dimensions. This is clear from the fact that the distance of 100,000 *li* belongs to a gnomon of 8 *chi* with a shadow length of 6 *chi* (see again Fig. 13.4). Since the calculation of the distance of the sun was wrong, as explained in the previous chapter, the calculation of the sun's diameter, which depends on it, must also be wrong.

However, there is something more fundamentally wrong with this measurement. In the *gai tian* system, the sun on the flat heaven orbits at a distance of 80,000 *li*

[12] Cf. Cullen (1996), 128.

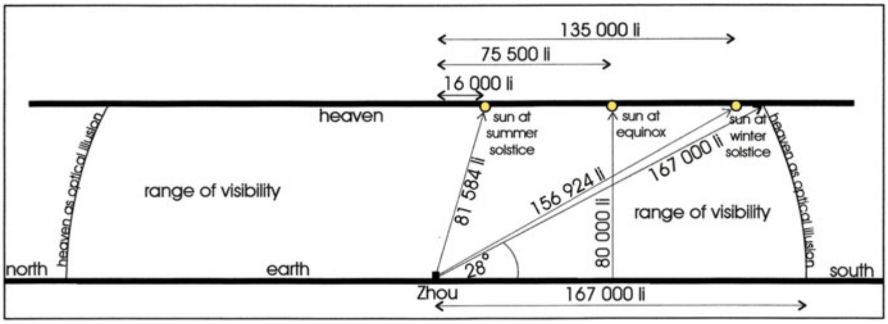

Fig. 14.3 Different oblique distances of the noon sun

above the flat earth. The sun's orbit in winter is much larger than in summer, and is situated, according to the *Zhou bi*, in spring and autumn exactly between these two, as shown in Fig. 13.6. As a result, the oblique distance from an observer in Zhou to the noon sun can vary enormously, from about $\sqrt{(16^2 + 80^2)} \times 1000 = 81{,}584$ *li* at noon during the summer solstice, to about $\sqrt{(135^2 + 80^2)} \times 1000 = 156{,}924$ *li* at noon during the winter solstice. For a person at a subsolar point, i.e. with the sun in his zenith, the oblique distance to the sun is equal to the vertical distance, which is only 80,000 *li*. In the eastern or western direction, the oblique distance of the sun is at its largest from an altitude of 28° and lower until sunrise and sunset, namely 167,000 *li*, which is the limit of the range of visibility (see Fig. 14.3).

We can learn from this analysis that the astronomers of the *Zhou bi* used their sighting tube of 80 cun with a bore of 1 cun in the situation of Fig. 13.4 (when the oblique distance to the sun equals 100,000 *li*) and not at other times of the day or year, because otherwise they would have discovered that they had to choose between two options, both of which would have undermined their ideas of a flat heaven over a flat earth. If they would have believed that the actual size of the sun is always the same, which they would probably have done, then they would have concluded that the sun was always at the same oblique distance from the observer, because in whatever direction and at whatever time of the day or year they would have looked at the sun, they would have been able to use exactly the same sighting tube with its dimension of 80 chi length and 1 cun diameter. This would have been contrary to the presuppositions of their *gai tian* system, according to which not the oblique distance from the observer, but the vertical distance from the earth is always and everywhere assumed to be the same.

On the other hand, if they were to adhere to the presuppositions of the *gai tian* system, they would have been confronted with the strange consequence that the size of the sun varied considerably with time, both during the day and throughout the year, and also with place. If the sun should always be seen from Zhou as having the same angular diameter, then the actual diameter of the sun should vary between 81,584 ÷ 80 = 1019.8 *li* at noon during the summer solstice and 167,000 ÷ 80 = 2087.5 *li* at sunrise or sunset on the horizon. In other words, the actual diameter of the sun on the horizon should be about twice as large as the sun in the zenith. This is purely the result of the system's conceptions and has nothing to do with the optical illusion that the sun seems to be bigger at the horizon, because if we use a sighting tube, that illusion disappears. Moreover, the consequence would have been that for an observer with the sun in his zenith the actual size of the sun would have been much smaller than the same sun seen from Zhou.

The Extension of the Solar Illumination

The *Zhou bi* provides a number of calculations that can be visualized in plan view drawings. Of course, the distances in these calculations and drawings do not have any relation to reality, because they are all ultimately based on the measurement of the sun's height, which was, in its turn, based on the wrong shadow rule. Whether or not the authors of the *Zhou bi* were aware of this, we do not know. Yet, these sections of the *Zhou bi* give the impression that they made their calculations not only to provide a better insight into the movements of the sun and the relation of its area of light to the range of visibility and the seasonal circles, but also just for fun. Anyway, I enjoyed drawing the pictures that visualize the calculations in the *Zhou bi*. We may readily surmise that the ancient Chinese astronomers made similar drawings to illustrate their calculations.

In the Figs. 14.4, 14.5, 14.6, 14.7, and 14.8, we must imagine the flat heavens projected on the surface of the flat earth. The distance from Zhou to the pole is therefore not the oblique distance, but the distance to the subpolar point on earth, and so on for all celestial points and circles. Zhou is indicated as a black square. The dotted circle is the range of visibility from Zhou. The yellow circle is the area of sunlight with the sun always in its center and rotating around the pole. I have added as black dots the points of the summer solstice noon sun, the spring/autumn equinox noon sun, and the winter solstice noon sun, all due south of Zhou. Characteristically, the *Zhou bi* also speaks of "the extent of solar illumination *at midnight* on the winter solstice," and "the extent of solar illumination *at midnight* on the summer solstice" (see texts 14.9 and 14.11 (#B28.1 and 2), quoted below). Accordingly, I added the points of the midnight winter sun, midnight spring/autumn sun, and midnight summer sun, all due north of Zhou. In Zhou, these midnight suns are invisible, because they are beyond the range of visibility of an observer in Zhou. The pole is also marked as a black dot, surrounded by the orbit of the *xuan ji* star. To understand the calculations, readers are requested to consult Figs. 13.6 and 13.18 in the previous chapter. All numbers in the next drawings must be multiplied by 1000. In the explanation of the drawings, I will use the following abbreviations for the distances we already became acquainted with in the previous pages:

AS = 167,000 *li* = the radius of the area of sunlight
ES = 59,500 *li* = the distance between the equator and the southern tropic
NE = 59,500 *li* = the distance between the northern tropic and the equator
NS = 119,000 *li* = the distance between the northern tropic and the southern tropic
PE = 178,500 *li* = the radius of the equator
PN = 119,000 *li* = the radius of the northern tropic
PS = 238,000 *li* = the radius of the southern tropic
PX = 11,500 *li* = the radius of the orbit of the *xuan ji* star
RV = 167,000 *li* = the radius of the range of visibility from Zhou
ZE = 75,500 *li* = the distance from Zhou to the equator
ZN = 16,000 *li* = the distance from Zhou to the northern tropic
ZP = 103,000 *li* = the distance from Zhou to the pole
ZS = 135,000 *li* = the distance from Zhou to the southern tropic
ZX = 91,500 *li* = the distance from Zhou to the orbit of the *xuan ji* star

Some calculations do not need extra drawings, for instance:

14.5. (#B21) [If one measures] south to the summer solstice noon and north to the winter solstice midnight, or south to the winter solstice noon and north to the summer solstice midnight, in both cases the diameter is 357,000 *li* and the circumference is 1,071,000 *li*.

The measurement is from the pole, so that we get: PN + PS or PS + PN; the circumference is calculated with $\pi = 3$.

The ancient Chinese astronomers were well aware of the fact that, as Zhao Shuang expresses it:

14.6. Beneath the north pole, the sun is in sight for 6 months, and out of sight for 6 months. For the 6 months from the spring to the autumn equinox[es] the sun is always in sight, while for the 6 months from the autumn to the spring equinoxes the sun is always out of sight. (. . .) What is called a year is a day and a night below the pole."[13]

The Figs. 14.4–14.8 not only show that they were acquainted with this fact, but also that they were able to account for it on a flat earth. In the previous chapter, we already saw that, quite surprisingly, the radius of the area of sunlight (AS) and the range of visibility (RV) was not given as 178,500 *li* and equal to the radius of the equator (PE), but as 167,000 *li*. The difference is due to a star, *xuan ji*, which allegedly describes a small circle with a radius of 11,500 *li* (PX) around the pole. In Fig. 14.4 is illustrated how the difference between the radius of the area of sunlight and the radius of the equator originates: PE − AS = PX.

[13]Quoted from Cullen (1996), 222.

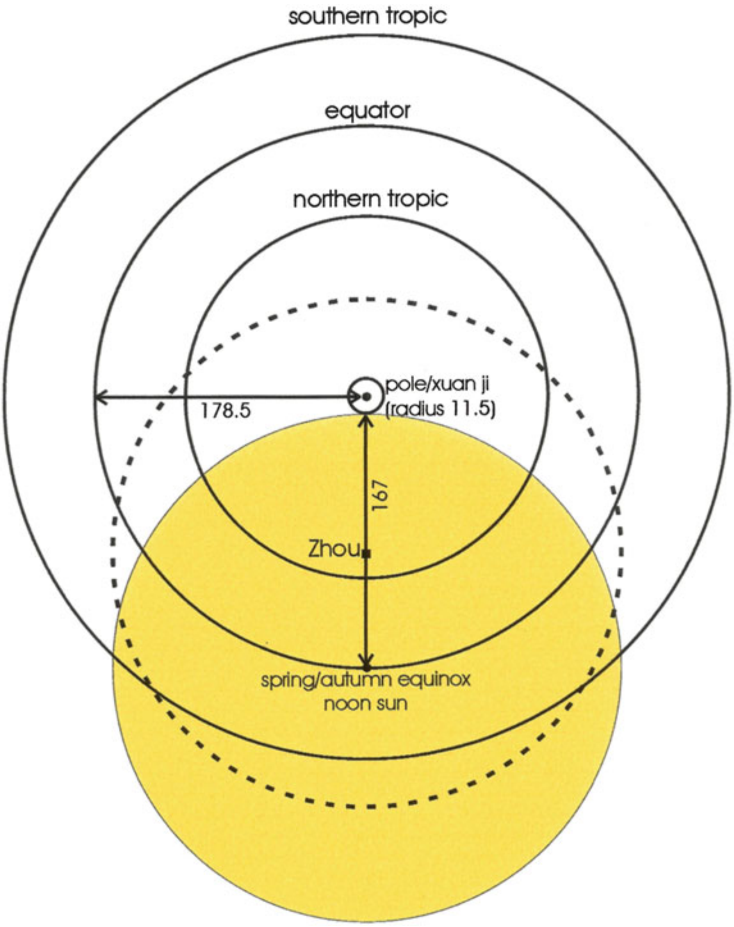

Fig. 14.4 The relation of the area of sunlight to the pole at the equinoxes. For a similar indication of limit the range of visibility from Zhou (the dotted line), see Cullen 2017, 211, Fig. 5.12 (there called: "range of sight" and "limit of sight")

During the summer solstice, the sun circles on the northern tropics and shines all day at the pole. The following relationships are calculated in the *Zhou bi* and shown in Fig. 14.5 as a result of Zhou's being situated 16,000 *li* north from the nearest point of the northern tropic (the distance ZN).

14.7. (#B26) At noon on the summer solstice the solar illumination extends 48,000 *li* south beyond the winter solstice noon. It extends 16,000 *li* south beyond the limit of human vision, 151,000 *li* north beyond Zhou and 48,000 *li* north beyond the pole.

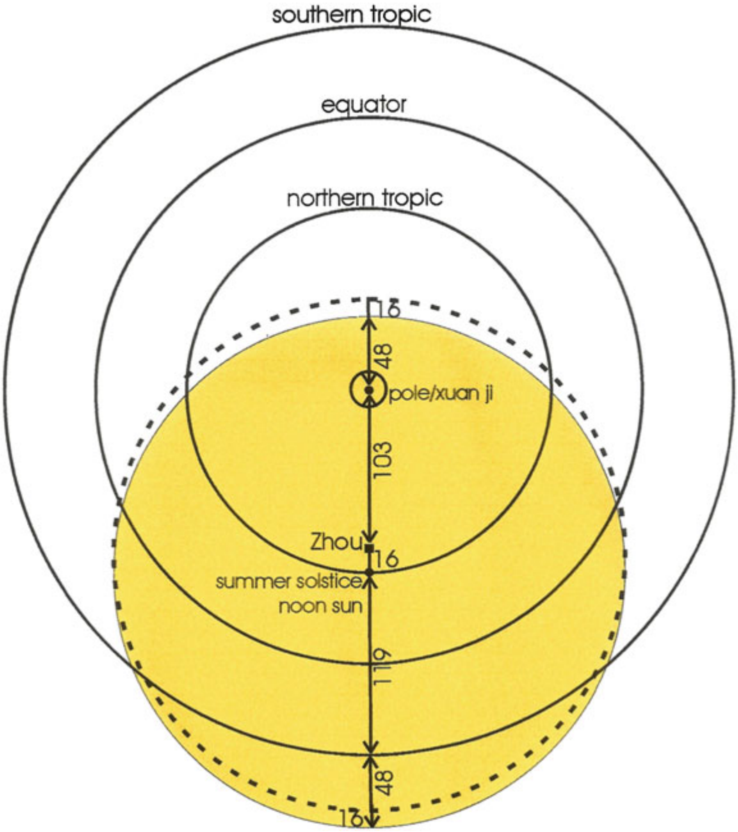

Fig. 14.5 Distances indicated in text 14.7 (#B26), at the summer solstice (some more measures added)

The calculations are: AS = NS + ZN + x → x = 48,000 *li*, AS = ZN + ZP + y → y = 48,000 *li*, and ZP + y = 151,000 *li*. Figure 14.5 also visualizes another calculation:

14.8. (#B25) The extent of vision from Zhou reaches 64,000 *li* north beyond the pole, and 32,000 li south beyond the winter solstice noon point.

Respectively: (48 + 16) × 1000, and (48 − 16) × 1000.

Figure 14.6 shows why we, when we are at Zhou, we can never see the sun when it is on the other side of the pole: even in summer, when the sun is at its nearest, the "midnight sun" will remain beyond our range of visibility and the area of sunlight will not reach Zhou. The number of 96,000 *li* is twice the value for y, found above.

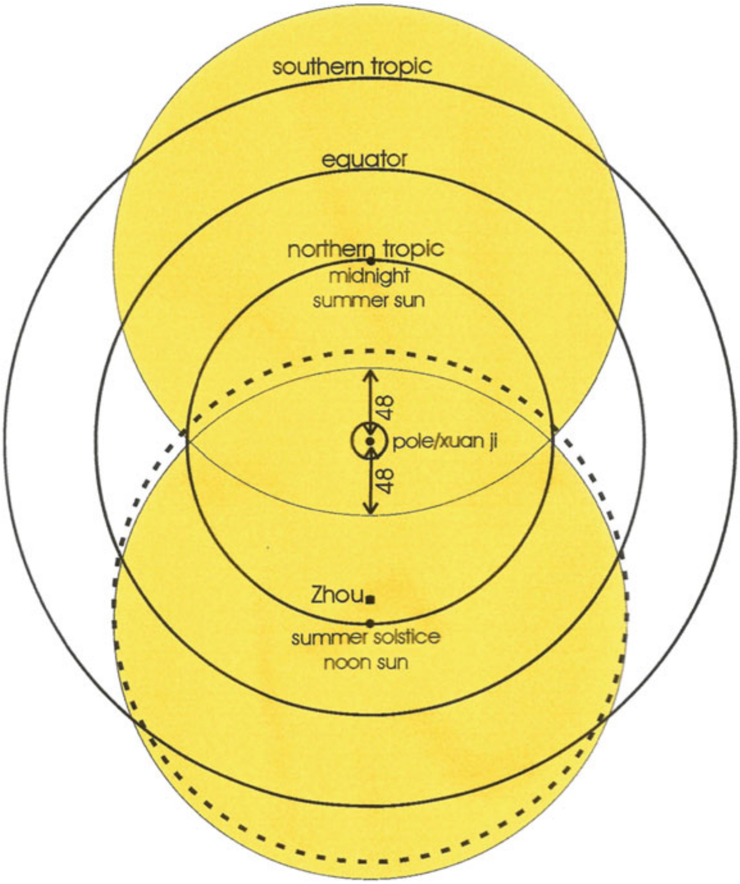

Fig. 14.6 Distances indicated in #B28.1 (text 14.9); the sun at noon and the "midnight sun" at the summer solstice

14.9. (#B28.1) At the summer solstice the illumination of the sun at noon and the illumination of the sun at midnight overlap by 96,000 *li* across the pole.

For the winter solstice, when it is night on the pole all day, the *Zhou bi* first gives calculations for the "midnight sun:"

14.10. (#B27) At midnight on the winter solstice the extent of solar illumination southwards falls short of the limit of vision of the human eye by 7000 *li*, and falls 71,000 *li* short of the subpolar point.

The number 7 in Fig. 14.7 we found already in the two previous drawings. The calculation is: PN − 48,000 *li* = 71,000 *li*; ZP + 71,000 *li* = 174,000 *li*, which is 7000 *li* more than RV.

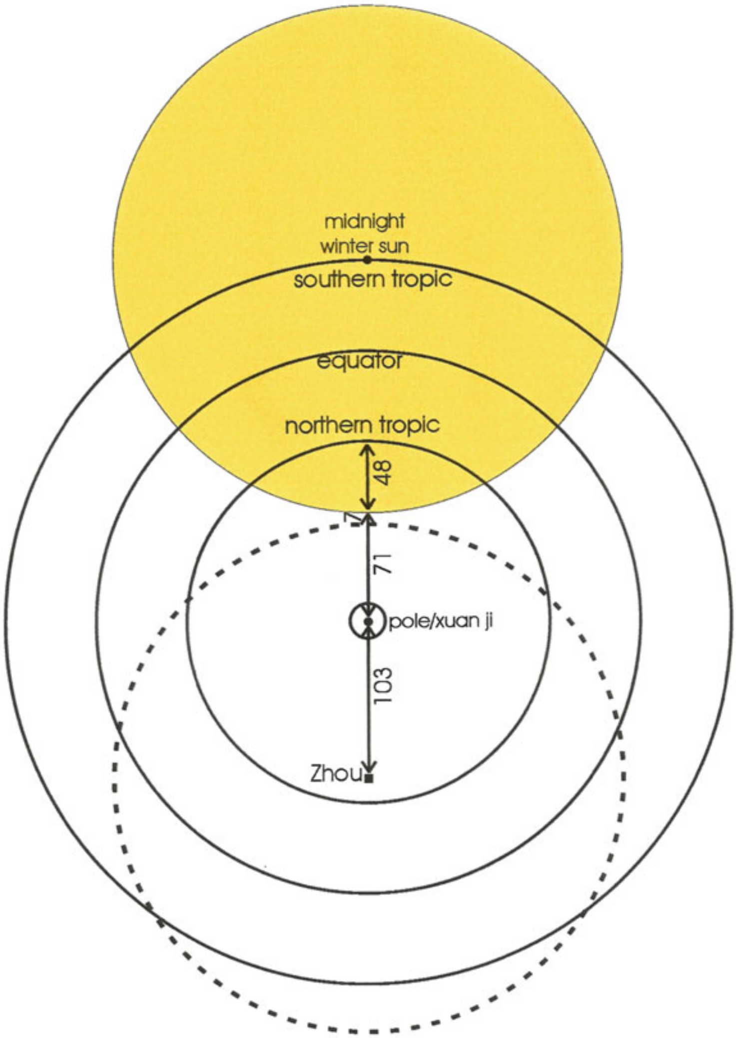

Fig. 14.7 Distances indicated in #B27 (text 14.10); the "midnight sun" at the winter solstice

And finally, the calculations for the sun at noon and the "midnight sun" at the winter solstice follow easily from the previous drawing, as shown in Fig. 14.8.

14.11. (#B28.2) At the winter solstice the illumination of the sun at noon falls 142,000 *li* short of meeting the illumination of the sun at midnight, and falls 71,000 *li* short of the subpolar point.

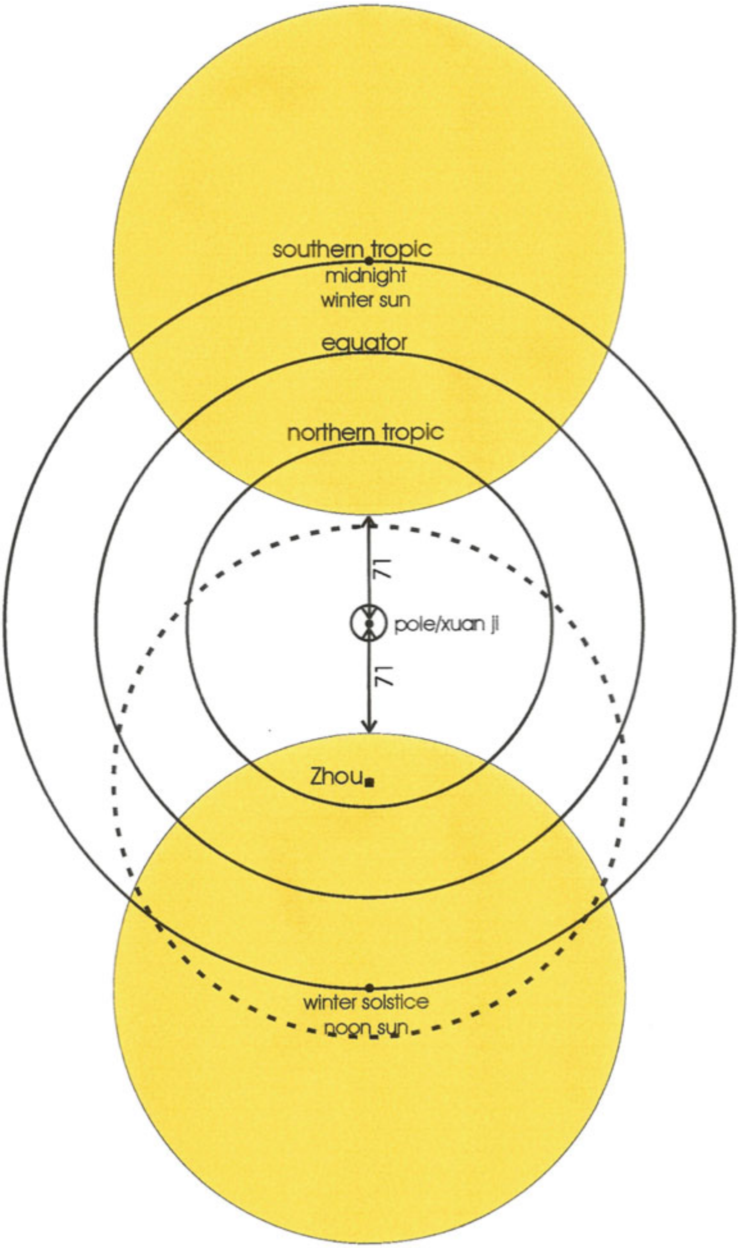

Fig. 14.8 Distances indicated in #B28/2 (text 14.11); the sun at noon and the "midnight sun" at the winter solstice

Geographical Measurements

This section deals with a number of geographical measurements and more specifically the distance from Zhou to some well-defined subsolar points, which are the points on earth where the sun is in the zenith. The calculations also illustrate the close relationship between cosmology and geography in the *gai tian* system. And again, we must bear in mind that the resulting geographical measurements are just as wrong as the celestial measurements, due to the wrong shadow rule. Rather than having any practical application, these calculations seem to have been made to demonstrate the possibility of producing all kinds of measurements within the *gai tian* system. The text of the *Zhou bi* only gives the outcomes of these calculations, but does not tell how they were achieved, and neither does Cullen. Below I will present them with the help of geometrical drawings. Because I used "E" and "W" for "East" and "West," the letters in these drawings do not always correspond to those of the list in the previous section. The lengths of the distances, however, can easily be checked with the list and with Figs. 13.4, 13.18, and 14.4, 14.5, 14.6, 14.7, and 14.8. In Figs. 14.9, 14.10, and 14.11, AB is the diameter of the tropics or the equator, AEB is a rectangular triangle (Thales' theorem), EZ is the perpendicular to the hypotenuse AB. Consequently, the triangles AEB, BZE, and AZE are all similar, so that AZ:

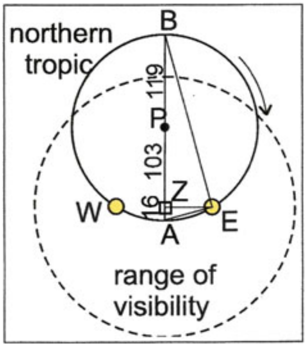

Fig. 14.9 Distance of the subsolar point E due east of Zhou at the summer solstice; see text 14.13. (#B29, first part)

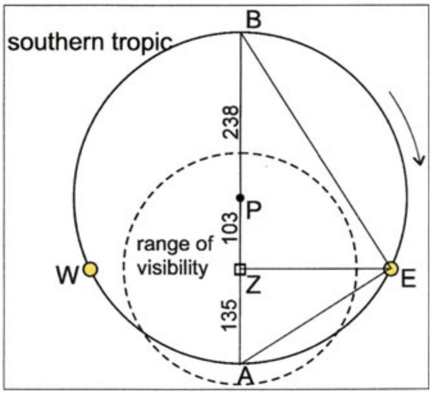

Fig. 14.10 Distance of the subsolar point E due east of Zhou at the winter solstice; see text 14.14. (#B29, second part)

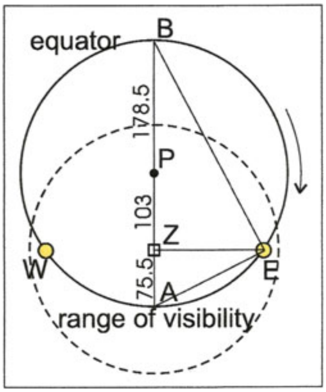

Fig. 14.11 Distance of the subsolar point E due east of Zhou at the equinoxes; see text on p. 308, at the end of this section

EZ = EZ: BZ → EZ^2 = AZ × BZ. Again, all numbers must be multiplied by 1000; the square Z is Zhou, the black dot P is the pole (i.e. the subpolar point).

Wang Chong remarks,

14.12. At the time when the sun sets in the west, the people living there will perhaps say that he is culminating, and looking from the point where the sun is setting, eastward to our world, heaven and earth may appear to the beholder joined together.[14]

The *Zhou bi* gives the calculations for the summer and winter solstices that belong to this remark.

14.13. (#B29, first part) On the day of the summer solstice, if one sights due east and west of Zhou then from the subsolar points directly due east and west of Zhou it is 59,595½ *li* to Zhou.

The calculation for the subsolar points due east and west of Zhou at the summer solstice goes as follows:

EZ^2 = AZ × BZ → ZE^2 = 3552 → ZE = 59.598,657,7 (×1000). The lengths AZ = 16, ZP = 103, and PB = 119 can be easily deduced from Fig. 13.6. As one can see in Fig. 14.9, the sun due east or west of Zhou is within the range of visibility of an observer at Z, as the *Zhou bi* says. Both this as well as the next two calculations indicate, by the way, that the Chinese astronomers recognized that there should not only exist places on the meridian where the sun could be observed right overhead (at a sub-solar point due south), but also elsewhere.

14.14. (#B29, second part) On the day of the winter solstices the sun is not visible in the regions due east and west, [however] by calculation we find that from the subsolar points it is 214,557½ *li* to Zhou.

[14]Forke (1907), 263–264.

For the lengths AZ = 135, ZP = 103, and PB = 238, see Fig. 13.6. The calculation for the subsolar points due east and west of Zhou at the winter solstice goes as follows: $ZE^2 = AZ \times BZ \rightarrow ZE^2 = 46{,}035 \rightarrow ZE = 214.557{,}684{,}6$ ($\times$ 1000). As one can see in Fig. 14.10, the sun due east or west of Zhou is beyond the range of visibility of an observer at Zhou, as the *Zhou bi* says.

The *Zhou bi* does not calculate the figures at the equinoxes, but when we do it in the same way as above, we get:

$EZ^2 = AZ \times BZ \rightarrow ZE^2 = 21{,}253.25 \rightarrow ZE = 145.784{,}944{,}4$ ($\times$ 1000). For the lengths AZ = 75.5, ZP = 103, and PB = 178.5, see Fig. 13.6. As one can see in Fig. 14.11, the sun due east or west of Zhou is within the range of visibility of an observer at Z. The consequence of this feature will be discussed in the next section.

Sunrise and Sunset Seen from Zhou

The teachings of the *Zhou bi* as explained in the previous section are not without problems. In Fig. 14.11, at the time of the equinox, the points W and E, representing the sun due east and west of Zhou, lie within the range of visibility of an observer at Zhou. Therefore, the sun must have risen further north-east of Zhou and set further north-west of Zhou, at the intersections of the circles of the equator and the range of visibility. This means that, according to the *gai tian* system, the day at the equinoxes in Zhou is shorter than the night, as can be seen in Fig. 14.12, where red is the part of the sun's orbit during the day and blue the part of its orbit at night. In reality, however, an observer at Zhou at the time of the equinoxes sees the sun rise due east and set due west, and the length of the day is equal to the length of the night. The wrong effect is due to the location of Zhou south of the subpolar point. This anomaly must have been the reason why the *Zhou bi* does not give a calculation for the

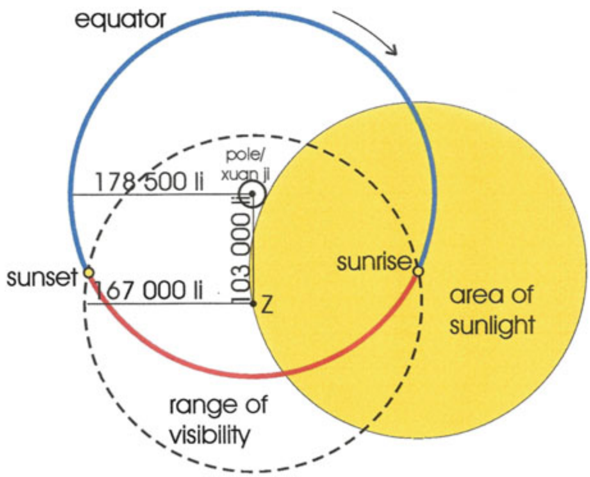

Fig. 14.12 Sunrise (and sunset) for an observer at Zhou at the equinox

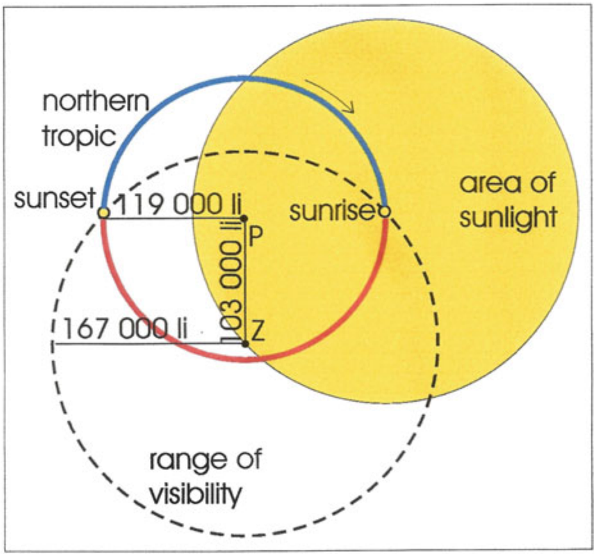

Fig. 14.13 Sunrise and sunset for an observer at Zhou at the summer solstice

equinoxes.[15] Perhaps it was also one of the reasons why the number of 167,000 *li* for the extension of the illumination of the sun, and thus also of the range of visibility was smuggled in: it weakens the wrong effect and makes the sun rise more to the east and set more to the west. If the range of visibility was much smaller than 167,000 *li*, namely 145,785 *li* (the distance ZE in Fig. 14.11), then, during the equinoxes, the sun would rise in Zhou due east and set due west. In that case, however, the difference in length between night and day would be greater.

In a fragment of his lost book *Xin Lun*, Huan Tan already made a similar point: the diagram drawn by his opponent Yang Ziyun could not be right, because the observer is south of the pole, and thus during the equinoxes his east-west line can never cut the circles of the orbit of the sun around the pole in two equal halves:

14.15. [Since heaven] is turning like a cover, that means that the northern [part of the sun's] track is distant from us, and the southern part is close. So how could the length of day and night be equal? Ziyun had no explanation.[16]

An analogous inconvenience appears with the summer solstice. As can be seen in Fig. 14.13, in the *gai tian* system at the summer solstice, when the sun is within the range of visibility of an observer at Zhou (the red half of the circle), the day appears to be about as long as the night, when the sun is beyond his range of visibility (the blue half of the circle). In reality, however, for an observer at Zhou during the summer solstice the day lasts much longer than the night, and he sees the sun rise in the north-east and set in the north-west.

[15] Cullen (1996), 131–132 makes the same point, although from his text it is not immediately clear that the wrong effect is due to the location of Zhou.

[16] Quoted from Cullen (1996), 60. See also Cullen (2017), 229, with Figure 5.15.

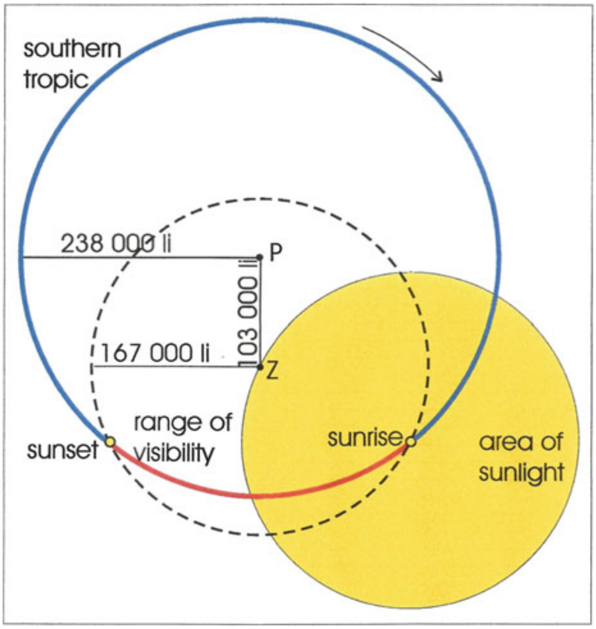

Fig. 14.14 Sunrise and sunset for an observer at Zhou at the winter solstice

At the winter solstice, an observer at Zhou will see the sun rise and set to the south of his east-west line, and the day is shorter than the night. This is also the case in the picture of the *gai tian* system, although the difference between the duration of day and night is clearly too great, and the day almost 3½ times as long as the night (see Fig. 14.14).

Of course, the anomalies shown in Figs. 14.12, 14.13, and 14.14 increase the further one goes south. Cullen also makes this remark with regard to the division between night and day at the equinoxes, without mentioning, however, the problems at the solstices.[17]

The Seven *Heng* and the Limit of the Cosmos

We have already seen that the circle nearest to the pole is that of the *xuan ji* star, with a diameter of 23,000 *li*. In the *Zhou bi* seven more concentric and equidistant circles are drawn around the pole, called *heng*, some of which we have already met: the first *heng* is the circle of the summer solstice (the northern tropic), the fourth *heng* is the circle of the equinoxes (the equator), and the seventh *heng* is the circle of the winter solstice (the southern tropic).

[17]See Cullen (1996), 132 and n. 153 in which he refers to a later Chinese critic, Wei Cheng.

14.16. (#D8) The first and innermost heng: diameter: 238,000 *li* (. . .).
14.17. (#D9) Next is the second *heng*: diameter: 277,666 *li* 200 *bu* (. . .).[18]
14.18. (#D10) Next is the third *heng*: diameter: 317,333 *li* 100 *bu* (. . .).
14.19. (#D11) Next is the fourth *heng*: diameter: 357,000 *li* (. . .).
14.20. (#D12) Next is the fifth *heng*: diameter: 396,666 *li* 200 *bu* (. . .).
14.21. (#D13) Next is the sixth *heng*: diameter: 436,333 *li* 100 *bu* (. . .).
14.22. (#D14) Next is the seventh *heng*: diameter: 476,000 *li* (. . .).

After these, one more circle is mentioned, the farthest extent of the area of sunlight. This circle is the ultimate boundary of the *gai tian* cosmos:

14.23. (#D15) Next comes the limit of solar illumination at the winter solstice. This goes 167,000 *li* beyond the outermost *heng*. This gives a diameter of 810,000 *li* (. . .).
14.24. (#D19) The diameter of the four poles is 810,000 *li* (. . .).
14.25. (#E3) Therefore the diameter of the outward extent of the sun's rays is 810,000 *li* (. . .).
14.26. (#B32) From the extent of the differences of the figures and the limit of solar illumination, the diameter of the four poles is 810,000 *li* (. . .).
14.27. (#D16) Nobody knows what is beyond this.

Since there is no sunlight beyond this ultimate limit, we might say that this is where the eternal night begins. A picture of these nine circles looks like Fig. 14.15;

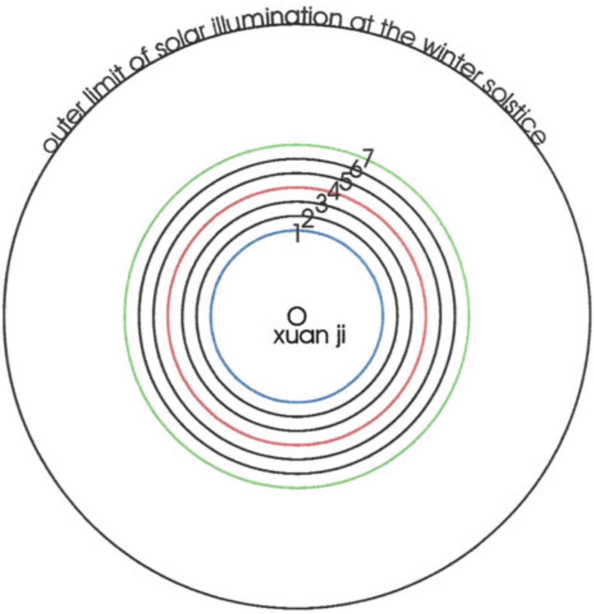

Fig. 14.15 The seven *heng*, the *xuan ji* circle, and the limit of solar illumination (For a similar picture, see Cullen 2017, 211, Figure 5.12)

[18]One *bu* = 6 *chi*, and 300 *bu* = one *li*, so one *bu* = 1.386 m (see also note 841).

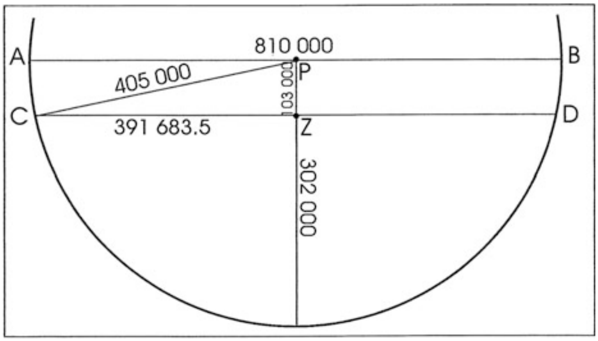

Fig. 14.16 The distance from Zhou to the outer limit of solar illumination

the blue circle (first *heng*) is the northern tropic, the red circle (fourth *heng*) is the equator, and the green (seventh *heng*) circle is the southern tropic.

The distance from Zhou to the outer limit of solar illumination can be calculated:

14.28. (#B33) From Zhou southwards to the [furthest]place illuminated by the sun is 302,000 *li*, and northwards to the [furthest] place illuminated is 508,000 *li* from Zhou. The distances east and west [from Zhou to the furthest points illuminated] are each 391,683½ *li*. Zhou is 103,000 *li* south of the center of heaven, and therefore the east-west measurement is shorter than the central diameter by just over 26,632 *li*.

The diameter AB is 810,000 *li*, according to texts 14.23 (#D15), 14.24 (#D19), 14.25 (#E3), and 14.26 (#B32); so, the radius CP is 405,000 *li*. The distance PZ is 103,000 *li*, according to texts 13.8 and 13.29 (#B14), and 14.28 (#B33), 13.30 (#F2), and 13.31 (#F4) (Fig. 14.16). Therefore, the distance from Z(hou) to the southern point of the circle AB is 405,000 − 103,000 = 302,000 *li*, and the distance from Z (hou) to the northern point of the circle AB is 405,000 + 103,000 = 508,000 *li*. The distance CZ is $\sqrt{(405^2 - 103^2)} \times 1000 = 391{,}683.5$ *li*. The distance CD is 2x CZ, which is 26,633 *li* shorter than AB.

An Extrapolation: The Southern Pole

The drawing of the seven *heng* does not tell the whole story of the implications arising from the logic of the model. The *gai tian* model displays regions similar to those on a spherical earth, as is shown in Fig. 14.17. There is a region where 2 days a year at noon the sun is in the zenith annually; this is the tropical zone between the two tropics. Between the northern tropic and the polar zone lies the temperate zone, in which China as well as the other ancient civilizations are situated. Note that the tropical zone (the zone between the two tropics) is twice as wide as the temperate

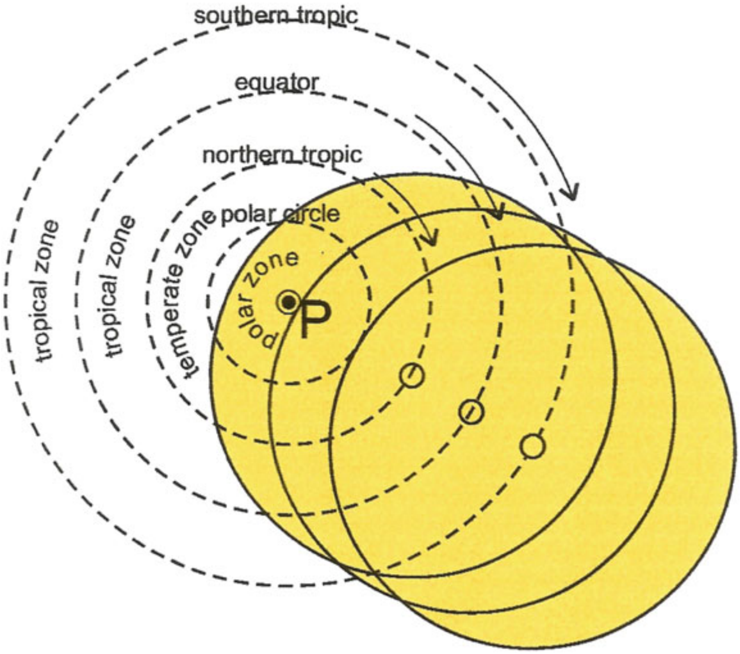

Fig. 14.17 The (northern) polar, temperate, and tropical zones

zone. The previous sentences presuppose that, next to the equator and the tropics, in the *gai tian* model there must also be a circle that corresponds to the arctic circle on a spherical earth. The area within the arctic circle is the polar zone. As can be seen in Fig. 14.17, the polar zone does not coincide with the area around the pole that corresponds with the orbit of the *xuan ji*. On a spherical earth, the arctic circle is defined as the periphery of an area around the north pole which would theoretically experience annually at least one 24-h period in which the sun is continuously above the horizon and at least one 24-h period in which the sun is continuously below the horizon.[19] However, we must not forget that the radius of the area of sunlight is assumed to be slightly smaller than that of the equator (167,000 *li* vs. 178,500 *li*). The definition of the polar circle is therefore not exactly applicable to the *gai tian* model. In Fig. 14.17, I have chosen to let the polar circle border the area of sunlight during the winter solstice.

Perhaps the strangest consequence of the model appears when we realize that there must be another temperate zone south of the southern tropic, bounded by another polar circle, beyond which there is another polar zone, and finally another,

[19]See *Wikipedia*, article “Polar circle.”

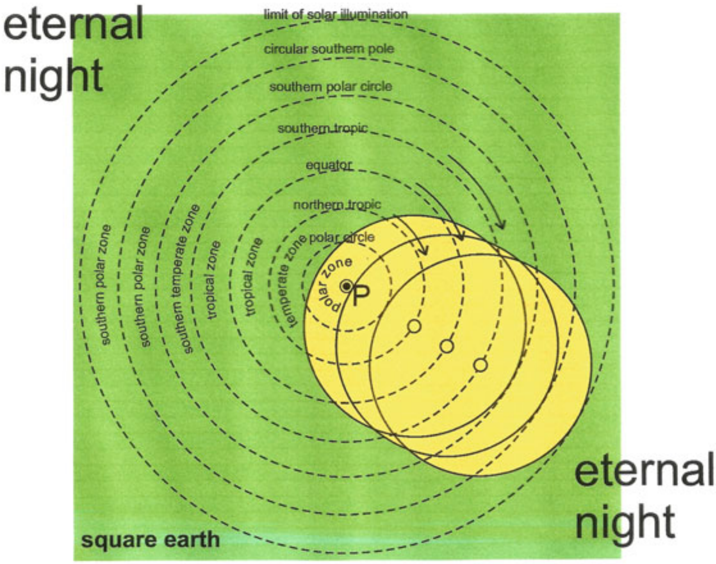

Fig. 14.18 The southern temperate and polar zones and the circular south pole

circular pole that can be called the south pole of the *gai tian* system. However, since in this system the days become shorter as one goes more southwards and the climate colder, the name "temperate zone" seems less suitable here. In this system, the south pole is not a point but a large circle at the edges of the earth. Moreover, at the outer limit of this circular south pole, which is also the limit of solar illumination, there is daylight only once a year, while it is night for the rest of the year.

As mentioned in the previous section, the *Zhou bi* uses the expression "the four poles" as an indication of the outer limit of solar light. Obviously, the idea is that not only the one central pole exists, but also four peripheral poles, which should be understood as the farthest places east, west, north and south at the outer limit of sunlight. Taking into account that for every observer, wherever on earth, the direction to the central pole is "north" and the opposite direction "south," it is less confusing to speak of one circular south pole instead of "the four poles."

The idea of southern circles and zones is rendered in Fig. 14.18 on the contours of a square earth. This picture also shows, incidentally, how small, as compared to the surface of the earth as a whole, the Chinese astronomers who adhered to the *gai tian* must have imagined their own country, somewhere in the northern temperate zone. The outermost circle is the limit of the area that can lit by the sun, albeit only once a year. Beyond this circle it is eternally night.

Major argues that the image of the canopy of a chariot defines a square within a circle. The concept of a circle within a square, as in Fig. 14.18, is however confirmed

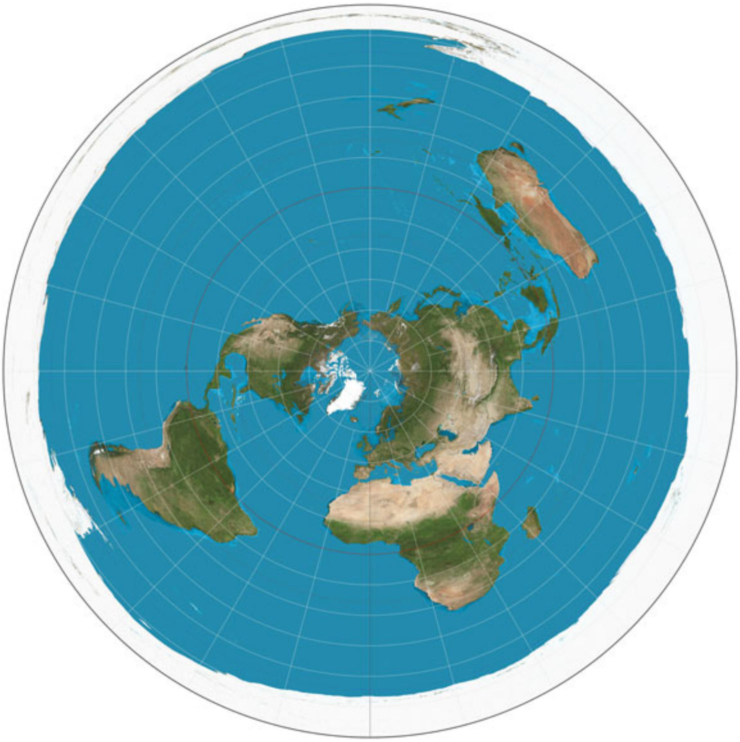

Fig. 14.19 A polar azimuthal equidistant projection of the earth (Courtesy Daniel R. Strebe, date listed for the image upload 15 August 2011)

by Shan Juli's critical remark to his teacher Zeng Shen (505–435 BC), that if heaven were round and earth square, then the four angles of the earth would not be well covered.[20]

The central north pole and the circular south pole are features that can be compared to those on a map of the spherical earth in a polar azimuthal equidistant projection of our spherical earth (see Fig. 14.19), although there it is an effect of mapmaking, whereas in the *gai tian* it is the result of the conception of heaven and earth.

How great was my surprise when I discovered that the same kind of projection is used in the emblem of the United Nations. The *gai tian* astronomers would have loved it (Fig. 14.20).

[20]Major (1993), 35 and figure 2.4 on 36. Shan Juli's remark quoted from Lan-ying Tseng (2011), 50.

Fig. 14.20 The emblem of the United Nations on the fence of the Geneva office

The Heaven Shaped Like a Truncated Conical Rain Hat?

There are some quite confusing phrases in section #E that are apparently inserted by a later editor, in which heaven and earth are no longer conceived as flat.

14.29. (#E6) Heaven resembles a covering rain-hat, while earth is patterned on an inverted pan.
14.30. (#E2) As for the subpolar point, it is 60,000 *li* higher than where humans live, and the pouring waters run down on all sides. Likewise the center of heaven is 60,000 *li* higher than its edges.
14.31. (#E8) Heaven is 18,000 *li* from earth. Even though the winter solstice sun is on the outer *heng*, it is still 20,000 *li* above the land below the pole.

These lines look like a kind of compromise between the *gai tian* and the *hun tian* systems, which fall outside the scope of this book, so I do not have to go into detail. As far as I can see it is impossible to keep the overall distance of 80,000 *li* intact when the earth is square and the heaven conical. This also remains a problem in Cullen's interpretation of these texts, in which he gives both heaven and earth the shape of a Japanese rain hat as a truncated cone.[21] Moreover, when the earth is shaped like an inverted pan, it obviously is no longer regarded as a square. And finally, "in a universe where heaven and earth are not flat and parallel, the shadow rule cannot rationally be applied," as Cullen remarks.[22]

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[21]See Cullen (1996), 135ff., and especially figure 13 on 136.

[22]Cullen (1996), 135. In Chap. 16, however, we will examine Cosmas Indicopleustes' limited version of the shadow rule, which is independent of the shape of the heaven, but only depends on Cosmas' assumption that the sun orbits parallel to the surface of the flat earth.

A Short Evaluation of the *Gai Tian* System in the *Zhou Bi*

At first glance, it may be tempting to judge that the *gai tian* system as a whole is of no value because it is fundamentally mistaken. Yet, the *gai tian* system must be considered as an impressive and unique conceptual construction. Let us enumerate once more its fundamental ideas: not only the earth but also the heaven is flat; the heavenly bodies describe circles overhead around the pole; rising and setting of the heavenly bodies are optical illusions; human sight is not infinite but limited; the sun throws a limited circle of light on the surface of the earth. It is amazing that with these innovations, all of which we would call erroneous, the early Chinese astronomers were able to achieve an acceptable picture of the universe, at least as seen from Zhou. The system makes it possible to measure not only the height of the heaven above the flat earth, but also many more distances in the universe.

Of course, based on suppositions we now know to be wrong, the system entails several serious problems. The picture of the sun orbiting overhead around the pole makes the sun's orbit in summer much smaller than in winter, and thus the summer sun's orbital velocity correspondingly slower. As a result of the introduction of the fictitious *xuan ji* star, the radius of the area of sunlight, and consequently that of the range of visibility is reduced to 167,000 *li* instead of 178,500 *li*. For an observer at Zhou, the calculations in the *Zhou bi* result an equinoctial day that is shorter than the night, while at the summer solstice the day appears to be about as long as the night. The more an observer goes in a southerly direction, the bigger this anomaly grows. An extrapolation of the *gai tian* model as a kind of azimuthal equidistant projection of the earth leads to unrealistic geographical dimensions, culminating in a circular south pole. The shadow rule used to measure the cosmological distances is obviously wrong and not based on observation. Because of a well-known optical illusion, the rising or setting sun appears larger than the sun high in the sky, but in the *Zhou bi* the measurement of the sun's diameter leads to an angular diameter of the sun as seen from a subsolar point that is bigger than from a point that is not directly below the sun. Moreover, more distant objects usually look smaller, which means that for an observer at Zhou the sun (and the moon) should seem smaller when they are farther away in their orbit around the pole. The *gai tian* also cannot explain why the setting sun is cut off at the horizon: the sun should become smaller, but not cut off. It remains unexplained that we can see stars at the horizon, at the limit of our visual field, although they are by far not as bright as the sun. It turned out that all heavenly bodies lower than 28° should be considered as optical illusions. Not only the rising and setting sun are illusions, but also the direction where their warmth comes from, as the story of Huan Tan and Yang Ziyun sitting on a porch shows. Even more important is that it remains strange that the sun and a part of the heaven, being beyond the range of visibility, are yet visible because of an optical illusion (a kind of fata morgana). The model cannot explain why from Zhou, being 103,000 *li* south of the pole, the orbits of the celestial bodies are seen as circles instead of ellipses. Perhaps one of Yang Xiong's objections points in the same direction: objection 8 points out that in the *gai tian* system, two stars should appear closer together when they move north of the pole than when they are in the south.[23] A serious problem is also that the system may be able to explain the occurrence of solar eclipses, it has difficulty explaining lunar eclipses.

In spite of all this, the *gai tian* system of heaven and earth that is presented in the *Zhou bi* is a highly sophisticated whole. It is not easy to fathom its basic concepts: a flat heaven over a flat earth, the range of visibility, the area of sunlight, the heaven and the rising and setting of heavenly bodies as optical illusions, the shadow rule, and the way they are all intertwined. The most extraordinary feature of the *gai tian* model is that it is able to account for the different times of the day on different parts of the earth. The authors of the *Zhou bi* were quite aware of this feature of their system, as evidenced by the way the movements of the sun and the limitation of the area if sunlight were described. When they spoke about the "midnight sun," these words were their way of expressing the idea of time differences all over the earth. It is my conviction that this feature was the very reason why the whole system was invented. As such it was an impressive intellectual achievement.

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Wikipedia, article "Polar circle."

[23]Quoted in *Sui shu* 19:4a, compiled by Wei Cheng and others, 636-656 BC Kind information by Christopher Cullen.